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Forced Convection Mass Transfer:

Part I. Effect of Turbulence Level on Mass Transfer through Boundary Layers with a Small Favorable Pressure Gradient

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The effect of free stream turbulence level on forced convection through laminar and turbulent boundary layers on a flat plate was studied in a wind tunnel with a small but nonzero favorable pressure gradient by means of the naphthalene sublimation technique.

With the small favorable pressure gradient used in this study, the rate of forced convection through laminar boundary layers agreed with Polhausen's theoretical equation for turbulence levels less than 2.8% but increased in a regular fashion for turbulence levels greater than 2.8% and was almost tripled at a free stream turbulence level of 11%. There was no evidence of an interaction between pressure gradient and turbulence level which would produce disproportionate effects on the rate of forced convection through laminar boundary layers. Substantially no effect of turbulence level on forced convection through turbulent boundary layers was observed for turbulence levels up to 7%.

The subject of forced convection through boundary layers on flat plates has been extensively studied, and the broad outlines are well known (*13, 14, 15*). When the partial pressure or concentration differences causing mass transfer are sufficiently small that the velocity normal to the surface can be neglected, the forced convection heat and mass transfer relations are identical, provided the appropriate form of the Stanton number is used and the Prandtl and Schmidt numbers are interchanged. However, there are some notable details which remain unresolved. Among these are the combined effects of turbulence level and pressure gradient on forced convection through laminar and turbulent boundary layers and to a lesser extent the effect of the value of the Prandtl (or Schmidt) number on the rate of forced convection through turbulent boundary layers. In general, the Stanton number is proportional to the $-\frac{1}{2}$ power of the Prandtl number for values of the Prandtl number near unity, to the $-2/3$ power for values from 10 to 100, and to the $-3/4$ power for very large Prandtl or Schmidt numbers (*15*).

Despite numerous investigations of forced convection through boundary layers (*11, 13*), it was not until the studies of Kestin et al. (*1, 12*) that conclusive evidence was presented showing that the coefficient of heat transfer from cylinders (32, 33) in cross flow and from flat plates was influenced in a different manner by the level of free stream turbulence. They showed that an increase in the turbulence intensity caused local changes in the heat transfer coefficients of cylinders (12), whereas the local effect was completely absent in the case of a flat plate (1) in zero pressure gradient flow for turbulence levels less than 3.8%. Qualitative studies (1) showed that the imposition of a favorable pressure gradient may have restored the local effect of free stream turbulence on convection through laminar boundary layers on flat plates, although the contribution of the effect of pressure gradient alone was not evaluated.

The effect of a pressure gradient on forced convection through laminar boundary layers in the absence of free stream turbulence has been analyzed for certain geometries. Eckert obtained an analytical solution for wedge flow (10). The velocity distribution was assumed to be an exponential function of the distance x measured from the stagnation point:

$$U_x = U_1 \, (x/L)^n \tag{1}$$

The result was

$$(N_{St})_x = \frac{K}{\sqrt{2-\beta}} \, Pr^{-1} \sqrt{U_x \, x/\nu} \tag{2}$$

where

$$K = 0.56 \, (\beta + 0.2)^{0.11} \, Pr^{0.35+0.02\beta} \tag{3}$$

and

$$\beta = 2n/(n+1) \tag{4}$$

Evans (16) obtained numerical solutions for forced convection through laminar boundary layers on flat plates for flows defined by

$$\frac{dU_o}{dx} = C U_o^{n'} \tag{5}$$

Extensive tables were presented which permit evaluation of the Stanton number for a wide range of Prandtl numbers and pressure gradients [as defined by Equation (5)].

The present studies are part of a program of studying the interaction of wakes from different geometrical shapes located away from surfaces with the fluid in the boundary layer immediately adjacent to the surface. The discovery of conditions under which significant interactions occur could have important implications for the development of turbulence promoters. This is particularly true for applications in which the turbulence promoters are required to have the common characteristic of neither blocking the heat or mass transfer surface nor having stagnant regions in which solids can accumulate.

In order to evaluate the performance of various promoters quantitatively, it was necessary to carefully establish the forced convection characteristics of the system in which the turbulence promoter studies were to be made. Thus the immediate objective of this study was the evaluation of the combined effect of turbulence level and pressure gradient on the rate of forced convection for the range of conditions of interest. Auxiliary tests were made to verify previous analytical (14, 20) and experimental (21, 23) starting length studies concerning the effect of a region of zero heat or mass transfer preceding the transfer surface on the rate of convective transfer.

The local rate of forced convection transfer through laminar and turbulent boundary layers was determined by measuring the difference in level of a flat naphthalene surface before and after exposure to an air stream, with procedures used similar to those described by Christian and Kezios (8) and Sherwood and Träss (4). Both the turbulence level of the free stream and the pressure gradient along the plate were measured in conjunction with the mass transfer studies.

EQUIPMENT AND PROCEDURE

The experiments were carried out in a once-through wind tunnel constructed of 1 in. thick plywood except for the experimental section which was made of 1 in. thick transparent plastic. Filtered air at a presure of 100 lb./sq. in. gauge was introduced into the wind tunnel through a jet injector (5) which had interchangeable critical flow nozzles to cover the flow rate ranges of interest. Additional air flow was induced by the jet injector from the room through a filter box. The jet injector was connected to the tunnel by a rubber bellows; the tunnel was rigidly attached to a steel frame which was supported on inflated rubber rings to isolate the tunnel from room vibrations. The natural frequency of the tunnel-frame assembly was approximately 800/min. based on measurements with an acoustic pickup and a frequency spectrum analyzer.

The air stream from the jet injector was expanded through a transition section with 34 deg. included angle; the transition section contained two concentric truncated cones to maintain uniform flow. The transition section was connected to a 4 ft. long by 20 in. sq. section. The first 13 in. contained a 4-in. section of aluminum honeycomb followed by four evenly spaced 24-mesh screens made from 0.011-in. diameter wire. The remaining 35 in. of this section served as a damping chamber. The flow area was then reduced to the final 3 × 12 in. cross section through a 7½ ft. long contraction section. The 6 ft. long test section discharged the air stream to the atmosphere.

The rate of mass transfer was determined by measuring the level of a naphthalene surface before and after exposure in the wind tunnel. The naphthalene was contained in a 6 × 17 × 1/16 in. recess in a 11 15/16 × 18 × 3/16 in. flat stainless steel plate. The leading edge of the plate was sharpened to a knife edge flat on the upper side with 11 deg. included angle; the plate was supported by four strips of metal 16½ in. long, 7/32 in. wide, and ½ in. high. The height of the plate and attached legs was constant to within ±0.0002 in. from front to back and from side to side, except for the recessed area.

The naphthalene surface was prepared by pouring molten naphthalene into the recess in the plate and allowing it to solidify. The surface was then smoothed with an electric iron. The result was a naphthalene surface which was flat to within ±0.008 in. from front to back and to within ±0.004 in. from side to side. [The surface of a plate may be considered (6) as aerodynamically smooth provided the roughness height is less than the value e_{ad} given by

$$\frac{U_o \, e_{ad}}{\nu} = 10^2 \tag{6}$$

This corresponds to a height of about 0.009 in. at a velocity of 20 ft./sec. and for the conditions used in these studies.]

Rigidly mounted dial-depth gauges sensitive to 0.0001 in. were used to measure the surface level after the steel plate was cooled to room temperature. The gauges were always zeroed against the stainless steel plate before a traverse was made; three traverses were made before and after each run, one down the center line, the other two ¾ in. from each side of the naphthalene layer. The duration of a given run was adjusted so that at least 0.002 in. and less than 0.015 in. of naphthalene sublimed. It is estimated (3, 7) that less than 0.00005 in. of naphthalene sublimed during the surface measurement.

The local mass transfer Stanton number corresponding to the difference in surface level caused by exposure of the flat plate in the wind tunnel is given by the expression

$$N_{St} = \frac{k_c}{U_o} = \frac{N_w \, P_{bar} \, M_A}{P_N \, \rho_A \, U_o \, M_N} \tag{7}$$

The difference in surface level was converted to mass flux N_w with a naphthalene density of 1.140 ± 0.026 determined from material removed from the test plate after a run and the duration of the exposure in the wind tunnel. The vapor pres-

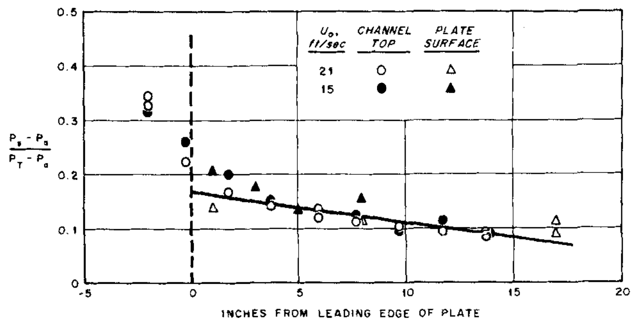

Fig. 1. Pressure gradient along the flat plate.

sure of naphthalene was calculated from the expression given by Sherwood and Bryant (*9*). Temperatures were measured with an iron-constantan thermocouple projecting into the free air stream ½ in. above the leading edge of the plate and projecting ½ in. in from the side wall of the test section. All runs were made at ambient temperature to minimize temperature variations along the naphthalene surface. The temperature depression of the mass transfer surface due to sublimation of naphthalene was estimated to be always less than 0.2°C. based on the relation derived by Sogin (*7*) which was experimentally verified by Träss (*3*). Schmidt numbers for the naphthalene-air system were calculated from the expression

$$N_{Sc} = 7(T_K)^{-0.185} \tag{8}$$

recommended by Sherwood and Träss (*4*). Equation (*8*) gives a value of 2.44 at 25°C.

Flow velocities were measured with a pitot tube for the range 3 to 100 ft./sec., with a calibrated vane type of anemometer for the range 2 to 10 ft./sec. and with smoke tracers from 0.2 to 3 ft./sec. The agreement of velocities measured by the different techniques was within ±3% when converted to a common basis (either mean or maximum velocity) in the range where the different techniques overlapped. The ratio of the maximum to mean velocity provided a sensitive measure of the region for transition from laminar to turbulent flow when the wind tunnel was considered as a closed conduit. The end of the laminar region occurred at a channel Reynolds number of 2,100, calculated from the hydraulic diameter. The transition region extended over a range of center-line velocities from 1.3 to 2.1 ft./sec.

AUXILIARY RESULTS

Pressure Gradient

The pressure gradient along the wind tunnel was measured at the top surface of the tunnel through ⅛-in. diameter holes spaced 2 in. apart along the center line and at the surface of a plate in the tunnel. The plate had the same dimensions as the mass transfer plate except that there was no recess in the top surface and there were 26 1/32-in. diameter pressure taps located along the center line of the plate. Static pressures $P_s - P_a$ were measured on the respective surfaces at the location of the holes by means of a micromanometer sensitive to 0.0005-in. water. The total pressure $P_t - P_a$ was measured at the same time as the static pressure measurement by means of a pitot tube 0.065-in. O.D. and 0.047-in. I.D. positioned at the center of the channel directly over the locations of the static pressure holes. The variation in pressure along the wind tunnel is shown in Figure 1 in terms of the pressure ratio:

$$P_r = \frac{P_s - P_a}{P_t - P_a} \tag{9}$$

as a function of distance from the leading edge for several different flow velocities. There was a small favorable pressure gradient which was almost linear with distance along the plate and so can be represented by the dimensionless parameter

$$\Lambda' = [L/(\rho_A U_o^2/2g_c)]\, dP_s/dx \tag{10}$$

The value of Λ' was 0.13 at the upper surface of the wind tunnel and 0.08 at the surface of the plate. An alternative method of indicating the magnitude of the pressure gradient is by expressing the velocity at the outer edge of the boundary layer as a function of distance along the plate by the following relation (*11*):

$$U_o = \text{const}\, x^m \tag{11}$$

Hartree's velocity profile shape factor β may then be calculated from the value of the exponent of Equation (11):

$$\beta = \frac{2m}{m+1} \tag{12}$$

The value of the center-line velocity increased about 10% in a regular fashion from the leading edge to the trailing edge of the plate giving a value for β of +0.09.

Velocity profiles were measured with a total pressure probe having a rectangular face with side dimensions of 0.030 and 0.19 in. attached to a traversing mechanism which could be read to 0.001 in .The static pressures were measured at a distance of ¼ in. from the plate. The distance from the plate was taken as the distance to the center of the probe. The laminar velocity profiles agreed well with the appropriate Hartree profiles (*11*). Turbulent velocity profiles agreed very well with a 1/7th power relation when the boundary-layer thickness was calculated from the relation

$$\delta = 0.37 \times (U_o x/\nu)^{-0.2} \tag{13}$$

Turbulence Level

The turbulence intensity

$$Tu = (\overline{u'^2})^{1/2}/U_o \tag{14}$$

was measured with a 'Lintronic" constant current hot wire anemometer with a 0.00014-in. diameter tungsten wire 0.25 in. long. Figure 2 shows the turbulence intensity as a function of center-line wind tunnel velocity. For some tests the turbulence level was increased markedly above that naturally occurring in the tunnel by placing a plate, perforated with either 1/16- or 1/8-in. diameter holes to give a fractional free area of 0.23 and 0.385, designated as "*A*" and "*B*", respectively, in front of the test plate. Although the turbulence generated by the perforated plates decayed rapidly, it was possible to obtain mass transfer data at turbulence levels as high as 11% by determining the rate of mass transfer directly beneath the location of the anemometer probe.

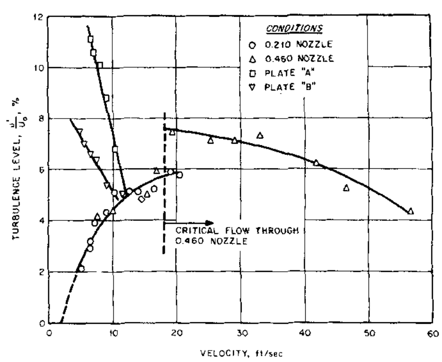

Figure 2. Turbulence intensity as a function of center-line velocity (plate A: 1/16-in. diameter holes, fractional free area = 0.23; plate B: ⅛-in. diameter holes, fractional free area = 0.385).

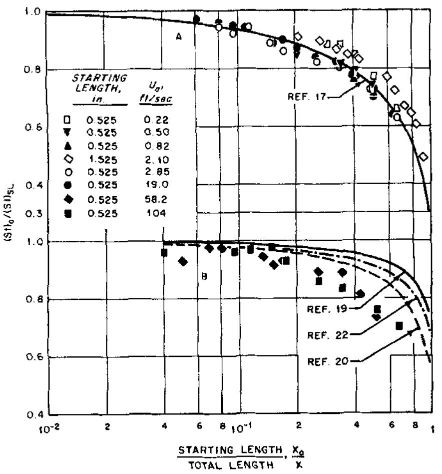

Fig. 3. Starting length correction factors: (A) laminar boundary layers, (B) turbulent boundary layers.

Starting Length

When the hydrodynamic boundary layer begins at the edge of the plate while forced convection starts at distance x_o from the leading edge, a correction must be made to obtain agreement with relations derived for forced convection taking place over the entire plate. The correction may be expressed as

$$\frac{(N_{St})_{SL}}{(N_{St})_o} = f\left(\frac{x_o}{x}\right) \tag{15}$$

The term $(N_{St})_{SL}$ refers to the Stanton number determined with a starting length of x_o, while N_{St} refers to Stanton numbers determined when the forced convection and hydrodynamic boundary layer both begin at the leading edge of the plate.

Analytical and empirical forms of the function $f(x_o/x)$ obtained by different investigators for laminar and turbulent boundary layers are given in Table 1. Data suitable for selecting one of the alternative forms for the turbulent boundary-layer correction factor are limited (*34*), and no data are known which confirm the validity of the laminar boundary-layer correction factor. In addition, no information was available on possible effects of turbulence level on either correction factor.

Experimental values of the function $f(x_o/x)$ are compared in Figure 3*a* and *b* with the functions given in Table 1. The laminar boundary-layer starting length correction factors are in good agreement with theory and show no effect of turbulence level over the range covered. The data for turbulent starting length correction factors are somewhat below the analytical expressions. This may be due to a short region of laminar boundary layer preceding the turbulent boundary layer. It was not possible in the present experiments to determine whether the turbulent boundary layer formed exactly at the beginning of the plate or developed in the first ¾ in. from the leading edge. There was no evidence of laminar or transition flow from ¾ in. on for the high velocity runs.

EXPERIMENTAL RESULTS

Local values of the Stanton number, calculated from Equation (7), corrected for starting length by Equation (15), are shown as a function of length Reynolds number in Figure 4. Although the bulk of these data are typical of those in which there was essentially no effect of turbulence level, one set of data (for 19.0 ft./sec. and turbulence level of 5.8%) is included to illustrate the

TABLE 1. STARTING LENGTH CORRECTION FACTORS $f(x_o/x)$

Type of boundary-layer flow	Basis*	$f(x_o/x)$, see Equation (15)	Reference
Laminar	A	$[1 - (x_o/x)^{3/4}]^{-1/3}$	*17*
Laminar	A	$[1 - (x_o/x)^{(3/2)(2-\beta)}]^{-1/3}$	*21*
Turbulent	A	$[1 - (x_o/x)^{39/40}]^{-7/39}$	*20*
Turbulent	E†	$[1 - (x_o/x)^{0.8}]^{-0.11}$	*22*
Turbulent	E**	$[1 - x_o/x]^{-0.09}$	*19*
Turbulent	E†	$1 + 0.40\,(x_o/x)^{2.75}$	*23*

* A = analytical; E = empirical.
† Derived from average Stanton number data.
** Obtained by differentiation of expression for average Stanton number.

pronounced effect of turbulence level on forced convection through laminar boundary layers which was observed in this study.

The laminar boundary-layer data at velocities less than 1 ft./sec. were obtained at zero turbulence level, since, as pointed out above, the flow in the wind tunnel channel was itself laminar. The data at a velocity of 2.85 ft./sec. were laminar over the complete length of the plate and agreed well with data for lower velocities despite the presence of a free-stream turbulence level of 0.8%. The experimental data are compared with theoretical lines

$$(N_{St})_x = K_1\,(N_{Re})_x^{-1/2} \tag{16}$$

where K_1 was calculated from Polhausen's analysis for zero pressure gradient and from Equations (1) to (4) and Evans' (*16*) tabulations for flow with a pressure gradient using the pressure gradient data shown in Figure 1. The results were K_1 = 0.1803 for zero pressure gradient, 0.1954, and 0.1954, respectively (*10, 16*), for the pressure gradient of this study.

The turbulent forced convection data shown in Figure 4 were calculated on the assumption that the boundary layer was turbulent over the entire length of the plate. The data from the region of the plate nearest the leading edge showed no indication of either laminar- or transition-region behavior. The turbulent curves were calculated from the theoretical relations of Deissler (*24*) and Spalding (*25*) and the empirical expressions of Zhukauskas and Ambrazyavichyus (*26*) and Colburn (*27*). Comparison of the data with the empirical and theoretical curves indicates that there was little effect of turbulence level on the rate of mass transfer for the particular pressure gradient of the present study.

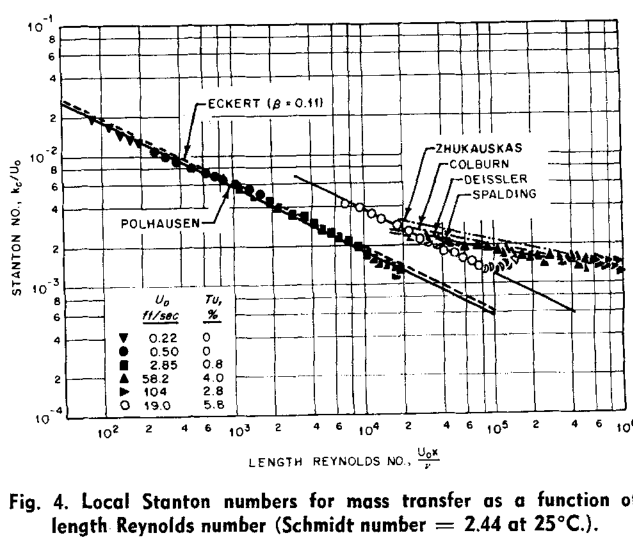

Fig. 4. Local Stanton numbers for mass transfer as a function of length Reynolds number (Schmidt number = 2.44 at 25°C.).

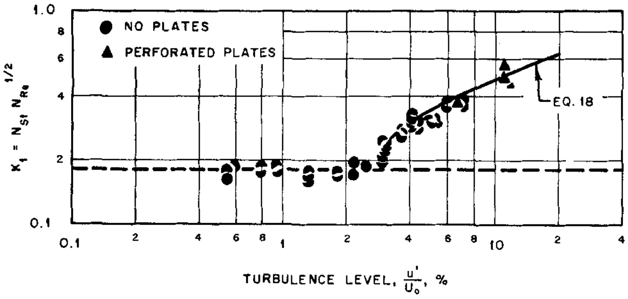

Fig. 5. Effect of turbulence level on the rate of mass transfer through laminar boundary layers ($\Lambda' = [L/(\rho_A U_0^2/2g_c)]dP_S/dx = 0.08$).

The marked increase in the rate of forced convection through laminar boundary layers caused by free stream turbulence levels greater than 2.8% is illustrated in Figure 5. In this figure the value of K_1 from Equation (16) is given as a function of the turbulence level as defined by Equation (14). The average value of Λ' [Equation (10)] was 0.13 for the free stream and 0.08 at the plate surface.

The free stream turbulence level was varied in two ways in order to prove that the observed increase in the rate of mass transfer was solely due to the turbulence level, either by varying the velocity over the range of 2 to 20 ft./sec. which produced the variation shown in Figure 2 or by placing perforated plates perpendicular to the flow ahead of the mass transfer plate. With the perforated plates the turbulence level at velocities in the range of 4 to 10 ft./sec. was increased by a factor of 2 or 3. The good agreement of the values of K_1 as function of turbulence level, independent of whether the turbulence level was the natural value associated with a certain velocity in the free wind tunnel or whether the turbulence was produced by the insertion of perforated plates, supports the belief that the observed effect was primarily due to the turbulence level in flows possessing a pressure gradient.

As is well known, the turbulence level of the free stream affects the point of transition of the boundary layer from laminar to turbulent flow. Gazley (*2*) summarized the results of many investigators, and his results are shown in Figure 6. Data from the present study on the transition point determined from the mass transfer data are in substantial agreement with Gazley's curves, as are the data from Kestin et al. (*2*) and Edwards and Furber (*31*). The curve labeled *laminar* refers to the Reynolds number for the last point on the laminar flow line on a plot of N_{St} vs. $(N_{Re})_x$, while the curve labeled *turbulent* refers to the Reynolds number for the first point on the turbulent curve on a plot of N_{St} vs. $(N_{Re})_x$.

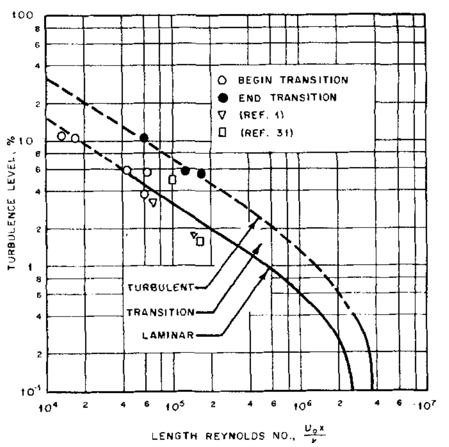

Fig. 6. Effect of turbulence level on the laminar-turbulent transition of mass transfer boundary layers.

DISCUSSION OF RESULTS

The present studies confirm the investigation of Kestin, Maeder, and Wang (*1*) in which it was shown that for moderate turbulence levels there was no effect of free stream turbulence on the rate of forced convection through a laminar boundary layer on a flat plate in the absence of a substantial pressure gradient. The studies of Kestin et al. (*1*) left several questions unresolved. Among these are the question whether in the absence of a substantial pressure gradient there is a threshold value for the turbulence level above which the turbulence level may influence forced convection, the question whether the combined effects of pressure gradient and free stream turbulence level are additive or whether there is an interaction of the two which produces a disproportionate effect, and the quantitative relation between turbulence level, pressure gradient, and rate-of-forced convection.

Theoretical answers to these questions are as yet unavailable. The usual analytical approach (*28, 29, 30*) is to study the behavior of laminar boundary layers when subjected to periodic oscillations in the free stream. Although available analyses are for frequency ranges far removed from regions of interest, they provide some insight into possible effects on the laminar boundary layer. For instance, they demonstrate that the primary effect may be interpreted as a secondary flow due to the interaction of inertial and viscous forces and further that this effect occurs in the region near the wall and not near the outer edge of the boundary layer. Unfortunately, the necessarily simplified procedures required to obtain solutions result in predictions of only small effects on the rate of forced convection and sometimes even indicate a decrease (*28*) rather than the observed increase.

The present study of the effect of free stream turbulence on the rate of forced convection through laminar boundary layers on a flat plate clearly shows, Figure 5, that there is a definite threshold value below which the turbulence level has little or no effect on the rate of forced convection when there is a small, favorable pressure gradient (that is, $\Lambda' \approx 0.1$). Above the threshold value (a turbulence level of ~2.8% in this study), the rate of mass transfer increased in a regular fashion and was almost tripled at a free stream turbulence level of 11%. Additional data are required to determine whether there is a relation between Λ' and the threshold level of turbulence or whether the threshold determined in this study is a constant for all pressure gradients.

When one assumes that there is no interaction between pressure gradient and turbulence level, one possible form for K_1 which is consistent with the experimental observations is

$$K_1 = k(\Lambda')[1 + f(Tu)] \quad (17)$$

where $k(\Lambda')$ may be determined from Equations (1) to (4) or from Evans' (*16*) numerical analysis. Combining Equations (1) to (4) with the data shown in Figure 5 permits the function $f(Tu)$ to be determined empirically. The result for the combined effects of pressure gradient and turbulence level is

$$K_1 = \left[\frac{0.56(\beta + 0.2)^{0.11}(N_{Sc})^{(0.35+0.02\beta)}}{\sqrt{2-\beta}\, N_{Sc}} \right] [1 + 0.60\,(Tu - 2.8)^{1/2}] \quad (18)$$

for $Tu > 2.8\%$, where it is understood that velocities normal to the surface are sufficiently small that the Prandtl or Schmidt number may be interchanged depending on whether heat or mass transfer is being considered. Equation (18) describes the present data with a maximum deviation of $\pm 12\%$.

The exploratory data of Kestin et al. (*1*) on the rate of heat transfer through laminar boundary layers on a flat plate with a pronounced pressure gradient are in reasonably good agreement with the values calculated from Equations (1) to (4) with a value of $\beta = 0.75$ estimated from the velocity profile data given in Figure 18 of reference *1*. This suggests that there is substantially no effect of turbulence levels of 1.1 to 1.7% on the rate of forced convection even in the presence of the large pressure gradient.

Although an expression of the general form of Equation (18) may be of general applicability in determining the combined effect of turbulence level and pressure gradient on forced convection through laminar boundary layers on flat plates, much additional experimental work is required to confirm the three empirical values in the last term on the right in Equation (18) in the absence of a suitable theoretical analysis.

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NOTATION

c_p = specific heat, B.t.u./lb.$_m$
C = coefficient [Equation (5)] ft.$^{-n'}$, sec.$^{n'-1}$
D = molecular diffusivity of mass, sq. ft./sec.
e_{ad} = roughness height, ft.
k = thermal conductivity, B.t.u./ft.-sec.
k_c = mass transfer coefficient, ft./sec.
$k(\Lambda')$ = coefficient [Equation (17)], dimensionless
K = coefficient [Equation (3)], dimensionless
K_1 = coefficient [Equation (16)], dimensionless
L = total length, ft.
m = exponent [Equation (11)], dimensionless
M = molecular weight, lb.$_m$/mole
n = exponent [Equation (4)], dimensionless
n' = exponent [Equation (5)], dimensionless
N_{Pr} = Prandtl number, $c_p\mu/k$, dimensionless
N_{Re} = Reynolds number, xU/ν, dimensionless
N_{Sc} = Schmidt number, ν/D, dimensionless
N_{St} = Stanton number, k_c/U_o, dimensionless
N_w = mass flux, lb.$_m$/sq. ft.-sec.
P = pressure, lb.$_f$/sq. ft.
Tu = turbulence level (%), dimensionless
T_K = temperature, °K.
u' = fluctuating velocity, ft./sec.
U_1 = velocity at the edge of the boundary layer where $x = L$, ft./sec.
U_o = free stream velocity
U_x = local velocity at the edge of the boundary layer, ft./sec.
x = length, ft.

Greek Letters

β = form factor, dimensionless
δ = thickness, ft.
ν = kinematic viscosity, sq. ft./sec.
ρ = density, lb.$_m$/cu. ft.
μ = viscosity, lb.$_m$/ft.-sec.
Λ' = form factor, dimensionless

Subscripts

A = air
a = atmosphere
N = naphthalene
o = free stream
r = reduced
s = static
SL = starting length
t = total

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